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Engineering Notes

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Accuracy Analysis of the Reflective Surface of the Umbrella-Type Antenna

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I. Introduction

IN MANY aerospace systems, the antenna requirements include large aperture sizes. One solution to this problem is the use of an umbrella-type reflector [1]that can be folded during launch and then opened to form a quasi-parabolic shape. Performance of an antenna, including the effect of spreading the focal point of the reflector into a focal region, depends critically on the precision of the membrane surface: that is, surface smoothness, deviation from desired surface profile, and slope error.

The ideal surface passing through all the ribs is the paraboloid of revolution. However, because the material of a umbrella-type reflector is flexible and elastic, the real working surface of a gore (the surface between any two adjacent ribs) has a saddle-shaped form; that is, it is convex along the meridian direction and concave in the circumferential direction. The degree to which the resulting surface conforms to a true paraboloid is obviously dependent upon the number of ribs; however, it is usually desirable to limit the number of ribs in the interests of minimizing weight. Significant deviations from a true parabolic shape, and consequent changes in the radiation characteristics may therefore be expected.

The root-mean-square (rms) error of the antenna working surface from the paraboloid of revolution is the adequate surface precision metric for the real antenna profile. This metric is widely used in the antenna community [2–8]. Agrawal et al. [9] obtained expressions for the rms error resulted by replacing a shallow spherical dish with an assemblage of flat facets in the shape of equilateral triangles, squares, and regular hexagons. Fichter [10] examined the potential for reducing rms error in shallow faceted reflectors by replacing flat facets with laterally curved membrane facets. Meyer [4] used linear membrane theory to derive expressions for rms error in faceted knitted-mesh reflectors of cylindrical parabolic or axisymmetric form.

In this study, the mathematical model of the reflective mesh of the umbrella-type antenna is discussed. The model of the reflective surface is considered for two extreme variants of the elastic parameters of the antenna surface. In the first case, the antenna membrane has zero stiffness in the meridian or radial direction, whereas in the second case, the antenna membrane has zero stiffness in the circumferential direction. The antenna reflector is usually treated as a membrane for which the deformations are described by the Laplace equation. However, in this study, it was possible to calculate the rms error without analysis of the antenna mode of deformation. The main contribution of this work is the determination of the limits of rms deviation of the reflective surface of the umbrella-type antenna from the best-fit paraboloid.

II. Theoretical Model and Analysis

Assume that the theoretic profile of the antenna shown in Fig. 1 is defined by the paraboloid equation as follows:

$$z = \frac{c}{r^2}(x^2 + y^2) \tag{1}$$

where c is the paraboloid height, and r is the outer radius of the paraboloid. Two radial rib stiffeners are placed on the paraboloid symmetrically with respect to the Oy axis so that the projections of stiffeners onto plane xOy intersect at the angle $\varphi = 2\pi/n$, where n is the number of ribs.

A paraboloid is a doubly convex surface; therefore, the best approximation is achieved in the case when the curvature of the antenna membrane in the circumferential direction is equal to zero. In other words, the ideal antenna membrane has zero stiffness along the meridian direction of the paraboloid. It is apparent that in this case, the real surface of the gore is formed by the line segments that connect ribs and are parallel to the planes xOy and xOz, as shown in Fig. 2.

The surface of the gore is the parabolic cylinder. The cross section of this cylinder by the plane xOz is the parabola described by the following equation:

$$z = ky^2 \tag{2}$$

where k is an unknown parameter. As illustrated in Fig. 1, $y = r\cos(\varphi/2)$ for $z = c$. Then the parameter k is defined as

$$k = \frac{c}{r^2 \cos^2(\varphi/2)} \tag{3}$$

Substituting Eq. (3) into Eq. (2), we obtain

$$z = \frac{c}{r^2 \cos^2(\varphi/2)} y^2 \tag{4}$$

Equation (4) defines the surface of the antenna gore for the case when the antenna membrane has zero stiffness in the radial direction. The antenna with such parameters is shown in Fig. 2.

Now define the rms error of the gore surface from the paraboloid using the following functional:

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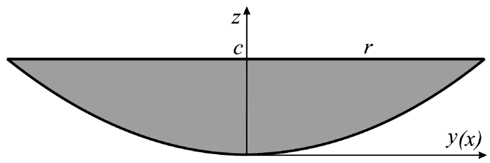
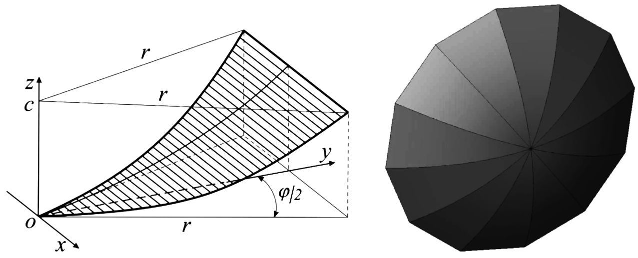


Fig. 1 Theoretical antenna profile.



a) Gore b) Reflector

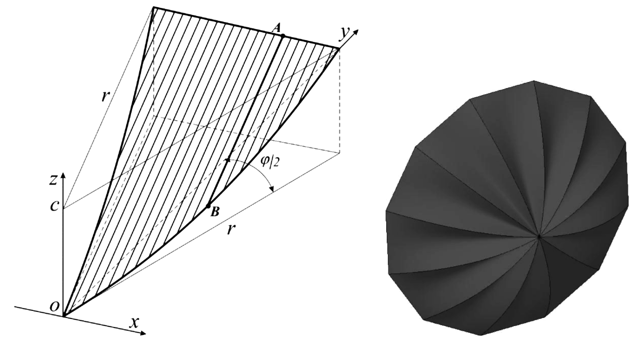
Fig. 2 Antenna with zero membrane stiffness in the radial direction.

$$w_1 = \sqrt{\frac{2}{S}\int_0^a \int_0^b \left[\frac{cy^2}{r^2\cos^2(\varphi/2)} - \frac{c}{r^2}(x^2+y^2)\right]^2 \mathrm{d}x\,\mathrm{d}y} \tag{5}$$

where $S = r^2 \sin(\varphi/2)\cos(\varphi/2)$ is the area of the gore projection on the plane xOy, $a = r\cos(\varphi/2)$, and $b = y\tan(\varphi/2)$. Integrating Eq. (5), we obtain

$$w_1 = \frac{2}{3}\frac{\sqrt{2}}{\sqrt{5}} c\sin^2\frac{\varphi}{2} \tag{6}$$

Equation (6) defines the rms deviation of the reflective mesh of the umbrella antenna from the paraboloid for the case when the antenna membrane has stiffness only in the circumferential direction. It is significant that this obtained value of w_1 is the limit and unimprovable rms error for the parabolic umbrella antenna.



a) Gore b) Reflector

Fig. 3 Antenna with zero membrane stiffness in the circumferential direction.

respectively. With reference to Fig. 3, we have

$$y_1 = r\cos\frac{\varphi}{2}, \qquad y_2 = x\tan^{-1}\frac{\varphi}{2}, \qquad z_1 = c$$
$$z_2 = c\frac{x^2}{r^2}\left(1 + \left[\tan^{-1}\frac{\varphi}{2}\right]^2\right) \tag{8}$$

Substituting Eq. (8) into Eq. (7), we obtain

$$z = c\left[1 + \frac{[y - r\cos(\varphi/2)][x + r\sin(\varphi/2)]}{r^2\sin(\varphi/2)\cos(\varphi/2)}\right] \tag{9}$$

Equation (9) describes the form of the antenna gore surface for the case when the antenna membrane has zero stiffness in the circumferential direction. The antenna with such elastic parameters of the membrane is shown in Fig. 3.

The rms deviation of the antenna gore surface [described by Eq. (9)] from the surface of the paraboloid [described by Eq. (1)] is given by the following functional:

$$w_2 = \sqrt{\frac{2}{S}\int_0^a \int_0^b \left[c\left(1 + \frac{[y - r\cos(\varphi/2)][x + r\sin(\varphi/2)]}{r^2\sin(\varphi/2)\cos(\varphi/2)}\right) - \frac{c}{r^2}(x^2+y^2)\right]^2 \mathrm{d}x\,\mathrm{d}y} \tag{10}$$

Now consider the second limit variant of elastic parameters of the antenna membrane. Assume that the material of the antenna membrane has zero stiffness in the circumferential direction. First, we connect the outer ends of the antenna ribs by the chord, as shown in Fig. 3. Then the surface of the antenna gore is formed by the line segments connecting the antenna ribs, with the chord and located in the planes parallel to the plane yOz, as shown in Fig. 3.

Now obtain the equation of the antenna gore surface. Because of the symmetry of the surface with respect to the plane yOz, we consider only half of the gore, i.e., the region characterized by $x > 0$. The canonical equation of an arbitrary line AB passing through two points has the following form:

$$\frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1} \tag{7}$$

where (y_1, z_1) and (y_2, z_2) are the coordinates of points A and B,

where $S = r^2 \sin\varphi/2\cos\varphi/2$ is the area of the projection of the gore surface to the plane xOy, $a = r\cos\varphi/2$, and $b = y\tan\varphi/2$. After rearrangement, we have

$$w_2 = \frac{c}{3\sqrt{10}}\sqrt{22 - 37\cos^2\frac{\varphi}{2} + 16\cos^4\frac{\varphi}{2}} \tag{11}$$

Equation (11) defines the maximum worst rms deviation of the gore surface from the paraboloid for the case when the antenna membrane has stiffness only in the radial direction.

Analyzing Eqs. (6) and (11) and taking into account that $\varphi/2 = \pi/n$, where n is the number of ribs, we can rewrite Eqs. (6) and (11) in the following form:

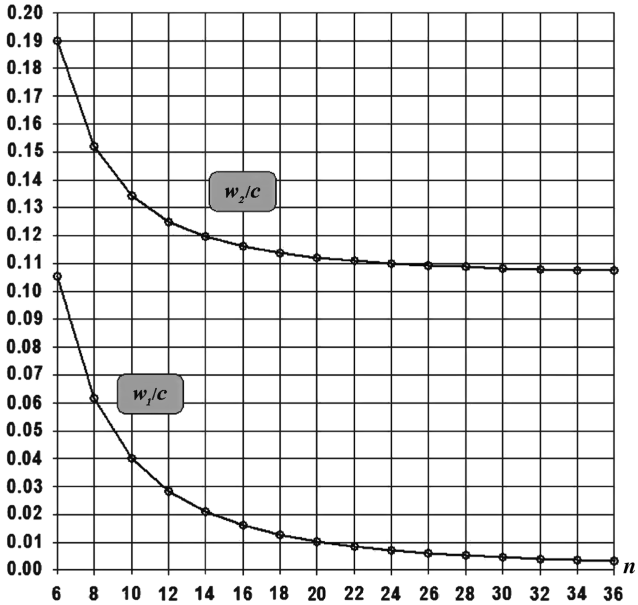


Fig. 4 Relative surface deviations w_1/c and w_2/c as functions of the number of ribs n.

$$w_1 = \frac{2}{3}\frac{\sqrt{2}}{\sqrt{5}} c \sin^2\frac{\pi}{n}$$
$$w_2 = \frac{c}{3\sqrt{10}}\sqrt{22 - 37\cos^2\frac{\pi}{n} + 16\cos^4\frac{\pi}{n}} \tag{12}$$

From Eq. (12), it follows that as $n \to \infty$, $w_1 \to 0$, whereas $w_2 \to c/(3\sqrt{10})$.

Figure 4 shows the influence of the number of ribs n on the values of w_1/c and w_2/c. Note that here, the number of ribs n is considered as a continuous variable.

III. Conclusions

The limits of variation of the rms error of the antenna surface from the paraboloid of revolution are defined in this study. The analysis of obtained results allows us to draw a conclusion about the necessary elastic parameters of the antenna membrane. The minimum rms deviation is obtained in the case when the material of the antenna membrane features well-defined anisotropy of the elastic properties. The warp yarns of the antenna membrane should be arranged in the circumferential direction, whereas weft yarns should be arranged in the radial direction. The obtained results can be used for the analysis of the deviations of the membrane surface at the stage of the preliminary design of umbrella-type antennas.

References

[1] Gryanik, M. B., and Loman, B. I., *Razvertyvaemye Zerkalnye Antenny Zontichnogo Tipa*, Radio i Svyaz, Moscow, 1987.

[2] Ruze, J., "Antenna Tolerance Theory:—A Review," *Proceedings of the IEEE*, Vol. 54, Apr. 1966, pp. 633–640.

[3] Zarghamee, M. S., "On Antenna Tolerance Theory," *IEEE Transactions on Antennas and Propagation*, Vol. 15, 1967, pp. 777–781.
doi:10.1109/TAP.1967.1139044

[4] Meyer, R. X., "Precision of Mesh-Type Reflectors for Large Space-Borne Antennas," *Journal of Spacecraft and Rockets*, Vol. 22, No. 1, 1985, pp. 80–84.

[5] Sokolov, B. N., "Optimizaciya Formy Poverhnosti Anizotropnoi Plenki," *Izvestiya Akademii Nauk SSSR*, No. 5, 1986, pp. 185–186.

[6] Baraov, A. H., and Sokolov, B. N., "O Predel'Noi Tochnosti Rabochei Poverhnosti Parabolicheskoi Antenny Zontichnogo Tipa," *Izvestiya Akademii Nauk SSSR*, No. 5, 1991, pp. 138–142.

[7] Banichuk, N. V., Karpov, I. N., and Klimov, D. P., *Mehanika Bol'Shih Kosmicheskih Konstrukcii*, Izdatel'Stvo Faktorial, Moscow, 1997.

[8] Jenkins, C. H., Wilkes, J. M., and Marker, D. K., "Improved Surface Accuracy of Precision Membrane Reflectors Through Adaptive Rim Control," *39th AIAA/ ASME/ ASCE/ AHS/ ASC Structures, Structural Dynamics, and Materials Conference*, AIAA, Reston, VA, 1998, pp. 2302–2308; also AIAA Paper 98-1983, Apr. 1998.

[9] Agrawal, P. K., Anderson, M. S., and Card, M. F., "Preliminary Design of Large Reflectors with Flat Facets," *IEEE Transactions on Antennas and Propagation*, Vol. 29, No. 4, 1981, pp. 688–694.
doi:10.1109/TAP.1981.1142631

[10] Fichter, W. B., "Reduction of Root-Mean-Square Error in Faceted Space Antennas," *AIAA Journal*, Vol. 22, No. 11, 1984, pp. 1679–1684.

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